$$= 1 - e^{-x}$$

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5. (iii) $C = 1 + x\cos\alpha + x^2\cos 2\alpha + \dots$ to ∞

$S = x\sin\alpha + x^2\sin 2\alpha + \dots$ to ∞

P1(4) - Maths paper-1st, Group-A Summation of series by C+iS method

$C+iS = 1 + x(\cos\alpha + i\sin\alpha) + x^2(\cos 2\alpha + i\sin 2\alpha) + \dots$

$xe^{i\alpha} = y$ (মনে) $= 1 + xe^{i\alpha} + x^2e^{2i\alpha} + x^3e^{3i\alpha} + \dots$

$= 1 + y + y^2 + y^3 + \dots = \dfrac{1}{1-y} = \dfrac{1}{1-xe^{i\alpha}}$

$= \dfrac{1}{1-x(\cos\alpha + i\sin\alpha)} = \dfrac{1}{(1-x\cos\alpha) - ix\sin\alpha}$

$= \dfrac{(1-x\cos\alpha) + ix\sin\alpha}{(1-x\cos\alpha)^2 + x^2\sin^2\alpha}$

$= \dfrac{(1-x\cos\alpha) + ix\sin\alpha}{1 - 2x\cos\alpha + x^2\cos^2\alpha + x^2\sin^2\alpha}$

Equating real parts.

$C = \dfrac{1-x\cos\alpha}{1-2x\cos\alpha + x^2(\cos^2\alpha + \sin^2\alpha)}$

$= \dfrac{1-x\cos\alpha}{1-2x\cos\alpha + x^2}$

5(i) $c = \cos\alpha + \frac{1}{2}\cos 2\alpha + \frac{1}{2^2}\cos 3\alpha + \frac{1}{2^3}\cos 4\alpha + \dots$ (1)

$s = \sin\alpha + \frac{1}{2}\sin 2\alpha + \frac{1}{2^2}\sin 3\alpha + \frac{1}{2^3}\sin 4\alpha + \dots$

$c + is = (\cos\alpha + i\sin\alpha) + \frac{1}{2}(\cos\alpha + i\sin 2\alpha) + \frac{1}{2^2}(\cos 3\alpha + i\sin 3\alpha) + \dots$

$c + is = e^{i\alpha} + \frac{1}{2}e^{i2\alpha} + \frac{1}{2^2}e^{3i\alpha} + \frac{1}{2^3}e^{4i\alpha} + \dots$

Let $x = e^{i\alpha}$.

$\therefore c + is = x + \frac{1}{2}x^2 + \frac{1}{2^2}x^3 + \frac{1}{2^3}x^4 + \dots$

$= x\left\{1 + \left(\frac{x}{2}\right) + \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^3 + \dots\right\}$

$= x \cdot \frac{1}{1 - x/2} = \frac{2x}{2x} = \frac{2e^{i\alpha}}{2 - e^{i\alpha}}$

$= \frac{2(\cos\alpha + i\sin\alpha)}{2 - (\cos\alpha + i\sin\alpha)}$

$= \frac{2(\cos\alpha + i\sin\alpha)}{\{(2 - \cos\alpha) - i\sin\alpha\}} \times \frac{(2 - \cos\alpha) + i\sin\alpha}{(2 - \cos\alpha) + i\sin\alpha}$

$= \frac{2(\cos\alpha + i\sin\alpha)(2 - \cos\alpha + i\sin\alpha)}{(2 - \cos\alpha)^2 + \sin^2\alpha}$

$= \frac{2(2\cos\alpha - \cos^2\alpha + i\sin\alpha\cos\alpha + i2\sin\alpha - i\sin\alpha\cos\alpha + i^2\sin^2\alpha}{(2 - \cos\alpha)^2 + \sin^2\alpha}$

$= \frac{2\{(2\cos\alpha - \cos^2\alpha - \sin^2\alpha) + i(\cancel{\sin\alpha\cos\alpha} + 2\sin\alpha - \sin\alpha\cos\alpha)}{4 - 4\cos\alpha + \cos^2\alpha + \sin^2\alpha}$

$$C+iS = \frac{2\left[2\cos\alpha - (\cos^2\alpha + \sin^2\alpha) + i\,2\sin\alpha\right]}{4+1-4\cos\alpha}$$ (2)

$$= \frac{2\cdot 2\cos\alpha - 2 + i\,4\sin\alpha}{5-4\cos\alpha}$$

$$= \frac{4\cos\alpha - 2}{5-4\cos\alpha} + i\,\frac{4\sin\alpha}{5-4\cos\alpha}$$

$$\therefore C = \frac{4\cos\alpha - 2}{5-4\cos\alpha}$$ Ans