

200814

$$= 1 - e^{-x}$$

P1(4) - Maths  
Paper - 1st, Group - A  
Summation of series  
by CTB method

$$S = 1 + x \cos x + x^2 \cos 2x + \dots \rightarrow \infty$$

$$S = x \sin x + x^2 \sin 2x + \dots \rightarrow \infty$$

$$CTB = 1 + x(\cos x + i \sin x) + x^2(\cos 2x + i \sin 2x) + \dots$$

$$x e^{i\theta} = x(\cos \theta + i \sin \theta) = 1 + x e^{ix} + x^2 e^{i2x} + x^3 e^{i3x} + \dots$$

$$= 1 + y + y^2 + y^3 + \dots = \frac{1}{1-y} = \frac{1}{1-x e^{ix}}$$

$$= \frac{1}{1-x(\cos x + i \sin x)} = \frac{1}{(1-x \cos x) - i x \sin x}$$

$$= \frac{(1-x \cos x) + i x \sin x}{(1-x \cos x)^2 + x^2 \sin^2 x}$$

$$= \frac{(1-x \cos x) + i x \sin x}{1 - 2x \cos x + x^2 \cos^2 x + x^2 \sin^2 x}$$

$$= \frac{(1-x \cos x) + i x \sin x}{1 - 2x \cos x + x^2}$$

$$1 - 2x \cos x + x^2 \cos^2 x + x^2 \sin^2 x$$

Equating real parts.

$$C = \frac{1 - x \cos x}{1 - 2x \cos x + x^2}$$

$$= \frac{1 - x \cos x}{1 - 2x \cos x + x^2}$$

$$= \frac{1 - x \cos x}{1 - 2x \cos x + x^2}$$

$$5(i) \quad e^z = \cos z + \frac{1}{2} \cos 2z + \frac{1}{2!} \cos 3z + \frac{1}{2^3} \cos 4z + \dots \quad (1)$$

$$s = \sin z + \frac{1}{2} \sin 2z + \frac{1}{2!} \sin 3z + \frac{1}{2^3} \sin 4z + \dots$$

$$c + i s = (\cos z + i \sin z) + \frac{1}{2} (\cos 2z + i \sin 2z) + \frac{1}{2!} (\cos 3z + i \sin 3z) + \dots$$

$$c + i s = e^{iz} + \frac{1}{2} e^{i2z} + \frac{1}{2!} e^{i3z} + \frac{1}{2^3} e^{i4z} + \dots$$

$$\text{let } x = e^{iz}$$

$$\therefore c + i s = x + \frac{1}{2} x^2 + \frac{1}{2!} x^3 + \frac{1}{2^3} x^4 + \dots$$

$$= x \left\{ 1 + \left(\frac{x}{2}\right) + \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^3 + \dots \right\}$$

$$= x \cdot \frac{1}{1 - \frac{x}{2}} = \frac{2x}{2 - x} = \frac{2e^{iz}}{2 - e^{iz}}$$

$$= \frac{2(\cos z + i \sin z)}{2 - (\cos z + i \sin z)}$$

$$= \frac{2(\cos z + i \sin z)}{(2 - \cos z) + i \sin z} \times \frac{(2 - \cos z) - i \sin z}{(2 - \cos z) - i \sin z}$$

$$= \frac{2(\cos z + i \sin z)(2 - \cos z + i \sin z)}{(2 - \cos z)^2 + \sin^2 z}$$

$$= \frac{2(2\cos z - \cos^2 z + i \sin z \cos z + i 2\sin z - \sin^2 z)}{(2 - \cos z)^2 + \sin^2 z}$$

$$= \frac{2\{ (2\cos z - \cos^2 z - \sin^2 z) + i(\sin z \cos z + 2\sin z - \sin^2 z) \}}{4 - 4\cos z + \cos^2 z + \sin^2 z}$$

$$C+is = \frac{2 [2e^{i2\lambda} - (\cos 2\lambda + i \sin 2\lambda) + i^2 \sin 2\lambda]}{4+1-4\cos 2\lambda} \quad (2)$$

$$= \frac{2 \cdot 2e^{i2\lambda} - 2 + i4\sin 2\lambda}{5-4\cos 2\lambda}$$

$$= \frac{4e^{i2\lambda} - 2}{5-4\cos 2\lambda} + i \frac{4\sin 2\lambda}{5-4\cos 2\lambda}$$

$$\therefore c = \frac{4\cos 2\lambda - 2}{5-4\cos 2\lambda} \quad \text{Ans}$$